changes in hydromechanical and thermal conditions for flow over the cylinder in the presence of the deposit layer is also important. Changes in the surface state and form of the body over which flow occurs should also be noted (in the experiments performed the longitudinal axis of the cylinder increased by 6-16%) on the distribution of thermal resistance over cylinder perimeter.

NOTATION

d_s, D, characteristic dimensions of particles and cylinder diameter; R, thermal resistance; t, temperature; α, λ, heat liberation and thermal conductivity coefficients; δ, height of deposit layer; μ, mass flow rate concentration; ϕ, angle measured from frontal point of cylinder. Subscripts: s, solid component; w, value at wall; dep, value characterizing deposit; e, in, values on external and internal cylinder walls; f, dispersed flow; ef, effective value; i, local value of quantity.

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CALCULATION OF THE PROBABILITY OF PASSAGE OF GAS PARTICLES THROUGH A CIRCULAR CHANNEL WITH ABSORBING WALLS

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UDC 533.6.011.8

We consider the mass transport of a gas subject to free-molecular flow in a circular channel with absorbing walls. An analytical expression for the gas flow rate is obtained.

In the development of various vacuum devices it is necessary to be able to calculate the flux of gas through channels joining the operating cavities of the devices. The gas flow rate through a channel in problems of this kind is conveniently represented in terms of the probabilities of passage, return, and absorption (capture of molecules by the walls of the channel).

The presently existing analytical expressions for these probabilities, which are so crucial in technology, either do not take into account absorption of molecules by the lateral walls of the channel [1], or are correct only for long channels [2]. In the present paper we obtain an analytical expression free from these restrictions.

We consider two volumes V_1 and V_2 joined by a cylindrical channel of radius R and length H, in which there is steady, free-molecular gas flow (Fig. 1). Because the contributions of the fluxes from the volumes V_1 and V_2 to the total gas flux through the channel can be separated in this case, there is no loss of generality in assuming that the volume V_2 is a vacuum.

We make the following assumptions in formulating the problem. The velocities of the molecules entering the channel are described by a Maxwellian distribution function; when a molecule collides with the wall of the channel it is absorbed with probability α and is diffusely reflected with probability $\beta = 1 - \alpha$ (α depends on the nature of the surface and

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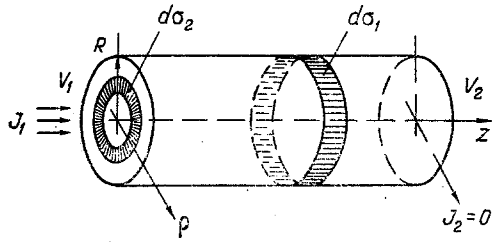

Fig. 1

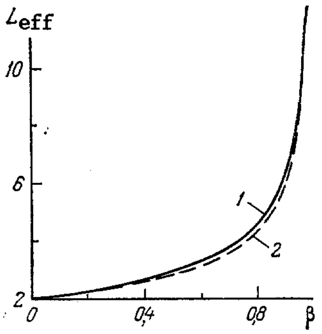

Fig. 2

Fig. 1. Formulation of the problem.

Fig. 2. Comparison of the exact and approximate expressions for L_{eff}: 1) approximate formula (9); 2) solution of the exact equation (8).

its temperature); the dependence of α on the excited internal degrees of freedom of the molecules is not taken into account, unlike the treatments in [3, 4]; collisions between the molecules inside the channel are neglected; the macroscopic parameters of the problem are distributed in an axisymmetric way.

Using the Lang boundary condition [5], and transforming from an integration in velocity space to an integration over the surfaces, we obtain a closed integral equation which generalizes the Clausing equation [6] to the case when there is absorption by the walls:

$$J(Y)=J_0(Y)+2\beta(Y)\int_0^1\frac{(Y^2+1-X^2)XYJ_1(X)\,dX}{(Y^4+2Y^2(1+X^2)+(1-X^2)^2)^{3/2}}+\frac{1}{2}\beta(Y)\int_0^L\left(1-|Y-Y'|\frac{(Y-Y')^2+6}{((Y-Y')^2+4)^{3/2}}\right)J(Y')\,dY', \tag{1}$$

where $X = \rho/R$; $Y = Z/R$; $L = H/R$; $J(Y)$, $J_0(Y)$, $J_1(Y)$ are respectively the number flux density of particles passing through an annular element of the lateral surface $d\sigma_1$, the intrinsic evaporation flux of this surface element, and the number flux of particles passing through the annular element $d\sigma_2$ on the end of the channel (see Fig. 1).

We will neglect the evaporation from the lateral surface ($J_0(Y) = 0$), which is correct if the temperature of the surface is not too high, and if the absorbed molecules interact strongly with the surface. We also neglect the possible change in the geometry of the channel because of absorption of molecules by its walls.

We now write down expressions for W_p, W_r, and W_a. Let F, F_r, F_p, F_a be the fluxes of molecules entering the channel from the volume V_1, leaving the channel into the volume V_1 and into the volume V_2, and absorbed by the walls of the channel, respectively. Then

$$W_p=F_p/F,\quad W_r=F_r/F,\quad W_a=F_a/F,\quad W_p+W_r+W_a=1. \tag{2}$$

Expressing F_p, F_r, and F in terms of J_1, J, and β we obtain

$$W_p=\left[2L^2\int_0^1\int_0^1\frac{(L^2+X^2+X'^2)XX'J_1(X')\,dX'dX}{(L^4+2L^2(X^2+X'^2)+(X^2-X'^2)^2)^{3/2}}+\right.$$

$$\left.+2\int_0^1\int_0^L\frac{((L-Y)^2+1-X^2)(L-Y)XJ(Y)\,dYdX}{((L-Y)^4+2(L-Y)^2(1+X^2)+(1-X^2)^2)^{3/2}}\right]\left[\int_0^1 XJ_1(X)\,dX\right]^{-1},$$

$$W_r=\left[2\int_0^1\int_0^L\frac{(Y^2+1-X^2)YXJ(Y)\,dYdX}{(Y^4+2Y^2(1+X^2)+(1-X^2)^2)^{3/2}}\right]\left[\int_0^1 XJ_1(X)\,dX\right]^{-1}, \tag{3}$$

$$J(Y)=2\beta(Y)\int_0^1\frac{(Y^2+1-X^2)YXJ_1(X)\,dX}{(Y^4+2Y^2(1+X^2)+(1-X^2)^2)^{3/2}}+\frac{1}{2}\beta(Y)\int_0^L\left(1-|Y-Y'|\frac{(Y-Y')^2+6}{((Y-Y')^2+4)^{3/2}}\right)J(Y')\,dY'.$$

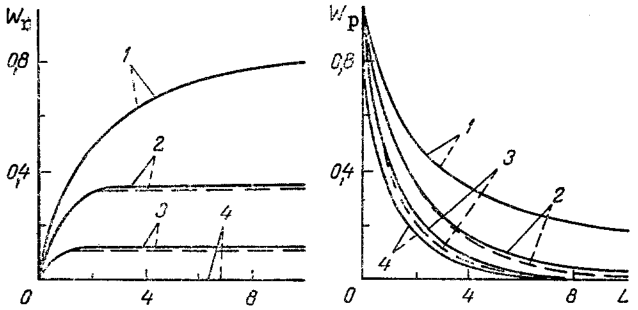

Fig. 3. Effect of the channel length L on the probability of passage W_p and return W_r for different values of β: 1) $\beta = 1.0$; 2) 0.8; 3) 0.4; 4) 0; solid curves: numerical results; dashed curves: calculated from (7).

The system of equations (3) was solved numerically to see whether a radial variation of the influx J_1 affects the probabilities W_p and W_r. The function $J_1(X)$ was taken to be a parabola ($J_1(X) \sim 1 + qX^2$). The absolute error in the calculations was 10^{-4} and it was assumed that $\beta(Y) = \text{const}$. The parameter q characterizing the nonuniformity of J_1 was varied from + 0.1 to − 0.1. The changes in W_p and W_r did not exceed 0.5% for all values of L and β.

The numerical analysis shows that one can assume $J_1(X) = \text{const}$ in the derivation of the analytical formulas for W_p and W_r. In addition, because the radial variation of J_1 does not affect W_p and W_r, one can model a channel in which $\beta(Y) \neq \text{const}$ as being composed of several sections with constant β. Then the radial distribution of the influx of the i-th section can be replaced by a constant proportional to the W_p of the preceding section (or W_r for reverse flow through the channel).

Then the system (3) can be rewritten in the form:

$$\begin{aligned} J(Y) &= \frac{\beta}{2}\left[\frac{Y^2+2}{(Y^2+4)^{1/2}} - Y\right] + \frac{\beta}{2}\int_0^L \left(1 - |Y - Y'|\,\frac{(Y-Y')^2+6}{((Y-Y')^2+4)^{3/2}}\right) J(Y')\,dY', \\ W_p &= 1 - \frac{L}{2}\left((L^2+4)^{1/2} - L\right) + \int_0^L \left[\frac{(L-Y)^2+2}{((L-Y)^2+4)^{1/2}} - L + Y\right] J(Y)\,dY, \\ W_r &= \int_0^L \left[\frac{Y^2+2}{(Y^2+4)^{1/2}} - Y\right] J(Y)\,dY. \end{aligned} \tag{4}$$

The system of equations (4) was solved by the Bubnov–Galerkin method [7]. The function J(Y) was modeled as

$$J(Y) = \begin{cases} b(L_{\text{eff}} - Y) & \text{for} \quad Y \leqslant L_{\text{eff}}, \\ 0 & \text{for} \quad Y \geqslant L_{\text{eff}}, \end{cases} \tag{5}$$

where $Y \in [0, L]$. The parameter L_{eff} characterizes the effective penetration depth of the molecules in the channel. It depends only on the probability of reflection β and can be much greater than the length of the channel. In this case only the first part of the expression for J(Y) in (5) need be used:

$$J(Y) = b(L_{\text{eff}} - Y), \qquad Y \in [0, L],$$

and in (7) below, ℓ will be the length of the channel L and not the effective length L_{eff}.

We introduce the notation:

$$A(X) = (X^2+4)^{1/2} - X, \quad B(X, Y) = XA(X) - (X - Y)A(X - Y),$$

$$C(X) = (8 - X^2)(X^2+4)^{1/2} + X^3 - 16,$$

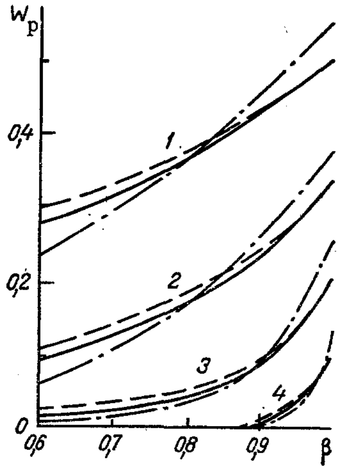

Fig. 4. Dependence of the probability of passage W_p on β for different channel lengths L: 1) L = 2.0; 2) 4.0; 3) 8.0; 4) 16.0; solid curves: calculated from [7], dashed curves: numerical results, dash-dotted curves: analytical expression from [2].

$$D(X) = 6X(X^2+4)^{1/2} - 24\ln\left|\frac{X+(X^2+4)^{1/2}}{2}\right|. \tag{6}$$

Then, using (5) and (6), we find

$$W_p = 1 - \frac{L}{2}A(L) + \frac{\beta A(L)B(L,\ l)}{8(1-\beta)+4\beta A(l)} - \\ -\beta C(l)\frac{3(2L-l)B(L,\ l) - 4[(L^2-2)A'(L) - ((L-l)^2-2)A(L-l) - 2l]}{24(1-\beta)l^3 + 12\beta D(l)},$$

$$W_r = \beta\left[\frac{2l - l^2A(l)}{4(1-\beta)+2\beta A(l)} + \frac{[C(l)]^2}{24(1-\beta)l^3+12\beta D(l)}\right], \tag{7}$$

where $\ell = \min(L, L_{eff})$, and $L = H/R$.

The quantity L_{eff} is determined by a complicated transcendental equation

$$\beta\left[1 - \frac{D(L_{eff}) - 2L_{eff}^3}{L_{eff}C(L_{eff})}\right] = \frac{2L_{eff}^2}{C(L_{eff})} - \frac{2(1-\beta)}{A(L_{eff})}, \tag{8}$$

however, an asymptotic analysis shows that L_{eff} can be calculated from the formula

$$L_{eff} \approx \frac{2}{\sqrt{1-\beta}} \tag{9}$$

with an error not exceeding 4% (see Fig. 2).

Graphs constructed from (7) are shown in Figs. 3 and 4. L_{eff} was calculated from (9). A graphical comparison with the analytical expressions from [1] was not done, because it can be easily shown that they are a special case of the result (7) when there is no absorption and β = 1. It is seen from Fig. 3 that the absolute error in the expressions in (7) does not exceed 0.01 over the entire range of L and β. The error reaches a maximum for L of the order of a few times L_{eff}. The function W_r demonstrates the physical meaning of L_{eff}: a molecule penetrating into the channel deeper than L_{eff} cannot return (the dependence $W_r(L)$ reaches a plateau for $L \sim L_{eff}$). When $L > L_{eff}$, the channel can be considered as infinite in the expression for W_r. The value of L_{eff} for a given β (the probability of scattering) can be used to determine the region of the channel which produces a returning gas flux through the intake cross section. Knowing L_{eff}, it is easy to determine the maximum returning flux, which is important in the design of high-vacuum devices. Comparison with [2] (see Fig. 4) demonstrates the superiority of the expressions (7). The formulas in [2] were derived assuming a long channel ($L \gg 1$) and they give significant errors when the channel length is not large.

Hence the results (7), while sufficiently simple, take into account the possibility of absorption of molecules by the lateral walls, and are applicable over the entire range of channel lengths and can therefore be used in engineering calculations.

NOTATION

V_1, V_2, gas volumes; R, radius of the channel; H, length of the channel; L, dimensionless channel length; ρ, radial coordinate; X, X', dimensionless radial coordinate; Z, longitudinal coordinate; Y, Y', dimensionless longitudinal coordinate; J, J_0, J_1, number flux densities; $d\sigma_1$, $d\sigma_2$, surface elements; α, probability that a molecule is absorbed in colliding with the wall; $\beta = 1 - \alpha$, probability that a molecule is diffusely reflected in colliding with the wall; W_p, W_r, W_a, probabilities of passage, return, and absorption; b, variational constant; q, ℓ, dimensionless parameters; L_{eff}, dimensionless effective penetration depth of the molecules in the channel; A, B, C, D, frequently used functions.

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EFFECT OF PERFORATION OF A RADIATION SCREEN ON PREEVACUATION OF EXTENDED CRYOGENIC SYSTEMS WITH HIGH-VACUUM THERMAL INSULATION

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UDC 533.5

Relations for determining the degree of perforation of the radiation screen as a function of the magnitude of gas evolution from the inner surfaces are obtained.

The traditional cryogenic system with high-vacuum thermal insulation usually consists of a vacuum chamber containing the object being cooled, surrounded by a radiation screen. The cooling process is preceded by preliminary evacuation of the vacuum volume. In those cases when the conductance of the evacuation channels in the chamber is quite high, the evacuation proceeds in the normal manner. In a number of cryogenic systems, however, the cross sections of the extended evacuation channels with a significant level of gas evolution, especially from the objects being cooled, are small. In this case large pressure drops (not only in the direction of evacuation) can occur in the chamber.

We shall study the process of preevacuation of such cryogenic systems for the model indicated in Fig. 1. In the calculations performed below it is assumed that gas evolution from the walls of the vacuum chamber and from the surface of the radiation screen is much weaker than gas evolution from the surface of the object being cooled and from the standard localized source at the far end of the chamber away from the pump.

These assumptions correspond to real cryogenic systems, where the walls of the chambers are usually made of materials with a low level of gas evolution. The object being cooled is itself often prepared with the use of polymers and the soldered and welded connections have large surfaces with a high level of gas evolution.

It should be noted that under real conditions it is virtually impossible to remove fluxes used in soldering from the surface and to remove different organic contaminants, which unavoidably remain after the preparation and assembly. Therefore, the magnitude of the gas evolution is determined not so much by the diffusion-desorption processes, occurring with molecules of gas dissolved in the bulk of the material or adsorbed on its surface, as by means of the saturated vapor pressure of different contaminants present on the object

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